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Technical Note 63

SINGLE SCATTERED NEUTRONS FROM AN ISOTROPIC POINT SOURCE

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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NATIONAL BUREAU OF STANDARDS
Technical Note

63

JULY 1960

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E. R. Mosburg, Jr., and W. M. Murphey

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Single Scattered Neutrons From an Isotropic Point Source

E. R. Mosburg, Jr., and W. M. Murphey

A calculation was made to determine the number of particles from an isotropic source which will reach a detector after scattering once in the medium surrounding the detector and source. The simple result obtained is applicable when the separation of the detector and the source is much less than the mean free path of the particle in the medium. The particular case of 14.1 Mev neutrons in air was considered. It was found that 44 percent of the single scattered neutrons had scattering angles of less than 30 degrees.

In a time-of-flight scattering cross section measurement using an isotropic source, the number of air scattered neutrons reaching the detector may be comparable to the number of neutrons from the scattering sample. Since rather thick shields are needed to appreciably reduce the background flux from the air scattered neutrons, it is desirable to have information about the angular distribution of the air scattered neutrons. The general problem of air scattering has been studied for various situations previously[1,2,3]. Here, for cases in which only the first scattering is important, a particularly simple form is obtained.

The number of neutrons reaching the detector from a specific infinitesimal volume element, dV, in space is given by the solid angle, g, subtended by the detector at dV, times the probability per unit volume, P, of scattering in the direction of the detector from the position of the volume element, times, n, the number of neutrons reaching the volume element from the source, times the size of the volume element, dV. This product is then integrated over all space, to give the total number, N, of neutrons reaching the detector;

$$N = \int gPn \, dV. \tag{1}$$

The probability per unit volume of a neutron scattering in the direction of the detector from the position of the volume element is

the cross section $\frac{\partial\sigma(\varphi)}{\partial\Omega}$ in millibarns per steradian as a function of scattering angle, φ, divided by the area element dA in cm^2 subtended by the volume element as seen from the source, times the number of target atoms per unit volume, n_t. So,

$$P = \frac{\partial\sigma(\varphi)}{\partial\Omega} \frac{1}{dA} n_t. \qquad (2)$$

For convenience the geometry as shown in fig. 1 will be used. All points with the same scattering angle lie on a circle drawn through the source, detector, and one of the scattering points with that angle (see note 1). The sum over all space will be carried out by using the cylindrical symmetry about the axis connecting the source and the detector and by integrating over all points with a common scattering angle and then integrating over scattering angle. Since there is cylindrical symmetry about the axis connecting the source and the detector the volume element, dV, can be defined as a ring of volume $dV = (2\pi r_1 \sin\theta_1) r_1 d\theta_1 dr_1$.

The solid angle, g, for a detector which is a sphere and has a cross sectional area of 1 cm^2 is $g = \frac{1}{r_2^2}$ steradians.

The number of neutrons reaching the volume element is $n = \frac{SdA}{4\pi r_1^2}$ where S is the total emission of the isotropic source in number per second. Since the total cross sections at 14.1 Mev for nitrogen and oxygen are about 1.5 barns (1 barn = 10^{-24} cm^2), and since there are about 5.3×10^{19} atoms per cm^3 in air, the mean free path of neutrons in air is about 86 meters. The exponential attenuation of the unscattered flux may be neglected for distances under about 10 meters.

1/ U. Fano, L. V. Spencer, and M. J. Berger, Penetration and Diffusion of X-rays, Encyclopedia of Physics Vol. 38/2 p.661.

2/ A. Langsdorf, Neutron Collimation and Shielding, Chapter IVE of Fast Neutron Physics, Part I by J. B. Marion and J. L. Fowler, Interscience Publishers (1960).

3/ R. E. Beissner and D. R. Smith, "Studies in Shielding", Convair Report FZK-9-117, 1958.

Making the above substitutions we have:

$$N = \int_V \frac{\partial\sigma(\varphi)}{\partial\Omega} \frac{1}{dA} n_t \, dV \frac{S}{4\pi r_1^2} \, dA \frac{1}{r_2^2} = \int_{r_1}\int_{\theta_1} \frac{\partial\sigma(\varphi)}{\partial\Omega} \frac{n_t S 2\pi r_1^2 \sin\theta dr_1 d\theta_1}{4\pi r_1^2 \, r_2^2}, \quad (3)$$

$$N = \int_{r_1}\int_{\theta_1} \frac{\partial\sigma(\varphi)}{\partial\Omega} \frac{n_t S \, \sin\theta_1 d\theta_1 dr_1}{2 \, r_2^2} . \quad (4)$$

From fig. 1:

$$\alpha = 180 - 2\,(\theta_1 + 90° - \varphi)$$

$$\text{so } \beta = 2\varphi - \alpha = 2\varphi - 180 + 2\theta_1 + 180° - 2\varphi = 2\theta_1 \quad (5)$$

$$\text{so } R \sin\theta_1 = \frac{r_2}{2} \,; \quad r_2^2 = 4R^2\sin^2\theta_1 .$$

$$N = \int_{r_1}\int_{\theta_1} \frac{\partial\sigma(\varphi)}{\partial\Omega} \frac{n_t S \, d\theta_1 dr_1}{8 \, R^2 \, \sin\theta_1} . \quad (6)$$

Now we transform from integrating over θ_1 and r_1 to integrating over θ_1 and φ.

$$d\theta_1 dr_1 = J\left(\frac{\theta_1 r_1}{\theta_1 \varphi}\right) \, d\theta_1 \, d\varphi \quad (7)$$

where

$$J\left(\frac{\theta_1 r_1}{\theta_1 \varphi}\right) = \begin{vmatrix} \dfrac{\partial\theta_1}{\partial\theta_1} & \dfrac{\partial r_1}{\partial\theta_1} \\ \dfrac{\partial\theta_1}{\partial\varphi} & \dfrac{\partial r_1}{\partial\varphi} \end{vmatrix} = \begin{vmatrix} 1 & \dfrac{\partial r_1}{\partial\theta_1} \\ 0 & \dfrac{\partial r_1}{\partial\varphi} \end{vmatrix} = \frac{\partial r_1}{\partial\varphi} \quad (8)$$

since θ_1 and φ are independent variables. Again from fig. 1 we have

$$r_1 = -\frac{D}{\sin\varphi} \sin(\theta_1 - \varphi), \tag{9}$$

where D is the distance from the source to the detector,

$$\frac{\partial r_1}{\partial \varphi} = D \frac{\sin\theta_1}{\sin^2\varphi}, \tag{10}$$

$$N = \int_{\varphi}\int_{\theta_1} \frac{\partial\sigma(\varphi)}{\partial\Omega} \frac{n_t S}{8 R^2} \frac{D}{\sin^2\varphi} d\theta_1 \, d\varphi . \tag{11}$$

Since $R = \dfrac{D}{2 \sin\varphi}$;

$$N = \frac{n_t S}{2 D} \int_{\varphi} \int_{\theta_1 = 0}^{\theta_1 = \varphi} \frac{\partial\sigma(\varphi)}{\partial\Omega} d\theta_1 d\varphi \tag{12}$$

Integrating over θ_1 we have the simple result,

$$N = \frac{n_t S}{2 D} \int \varphi \frac{\partial\sigma(\varphi)}{\partial\Omega} d\varphi . \tag{13}$$

This integral may be used for any single scattering where exponential attenuation of the number of particles may be neglected. Usually $\frac{\partial\sigma(\varphi)}{\partial\Omega}$ is not known as an analytic function and one must integrate numerically. Equation (13) actually gives the number of neutrons for one energy. If $\frac{\partial\sigma(\varphi)}{\partial\Omega}$ varies greatly over the energies of neutrons being considered then an integration of (13) must be made for each energy.

In our case N was determined for a 14.1 Mev neutron source. Since the total cross sections of nitrogen and oxygen are about equal at 14 Mev and exact results were not required it was assumed, for the purpose of $\sigma(\varphi)$, that the air was all nitrogen. D was 100 cm and the angular distribution (fig. 2) was that given in BNL-400[4/]. The results of the integration of (13) are plotted versus scattering angle, φ, in fig. 3. Fig. 4 shows the fraction of single scattering angle less than a given angle. Reading from the graph of fig. 4 one notices that 44 percent of the single scattered neutrons have scattering angles of less than 30 degrees. It was concluded, therefore, that the detector shielding for scattering angles less than 30° should be designed to give about three times as much attenuation as the shielding for other angles.

NOTE:

$$\varphi = \theta_1 + \theta_2$$

α is a constant

$$\theta_1 = (90 - \frac{\delta}{2}) - (90 - \frac{\alpha}{2})$$

$$\theta_1 = -\frac{\delta}{2} + \frac{\alpha}{2}$$

$$\theta_2 = 90 - (\frac{\alpha}{2} - \frac{\delta}{2}) - \left[90 - (\frac{\alpha}{2})\right]$$

$$\theta_2 = -\frac{\alpha}{2} + \frac{\alpha}{2} + \frac{\delta}{2} = +\frac{\delta}{2}$$

Therefore $\theta_1 + \theta_2 = -\frac{\delta}{2} = +\frac{\delta}{2} + \frac{\alpha}{2} =$ a constant.

Therefore φ is a constant independent of θ_1 and θ_2.

[4/] D. J. Hughes and R. S. Carter, Neutron Cross Sections, Angular Distributions. BNL 400, June 1956.

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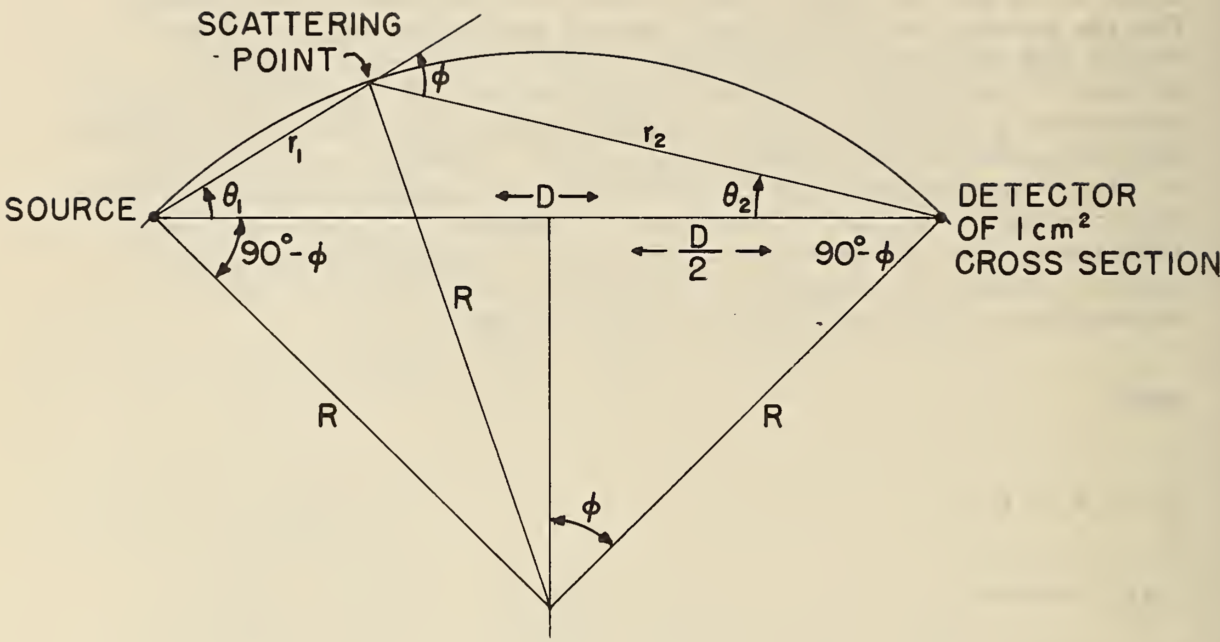

Figure 1. Geometry used in the calculation.

$$R^2 = R^2 + r_2^2 - 2r_2 R \cos(\theta_1 + 90 - \theta_2)$$

$$\varphi = \theta_1 + \theta_2$$

$$r_2 = 2R \sin \theta_1$$

$$R = \frac{D}{2 \sin \varphi}$$

r_1 = distance from source to scattering point

r_2 = distance from scattering point to detector

φ = scattering angle

R = radius of a circle through source, scattering point and detector.

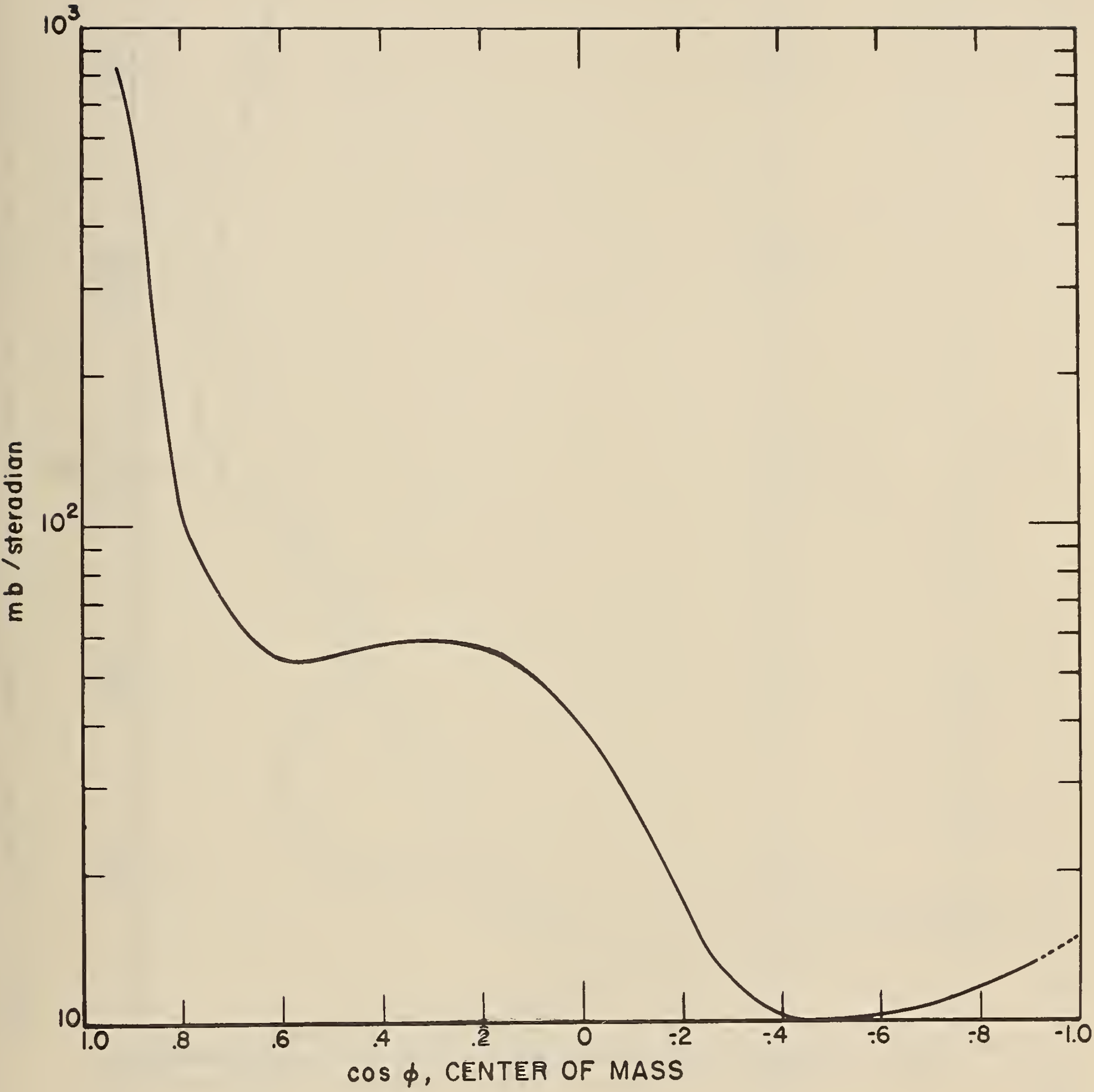

Figure 2. Elastic scattering cross section of 14.1 Mev neutrons on nitrogen (BNL-400).

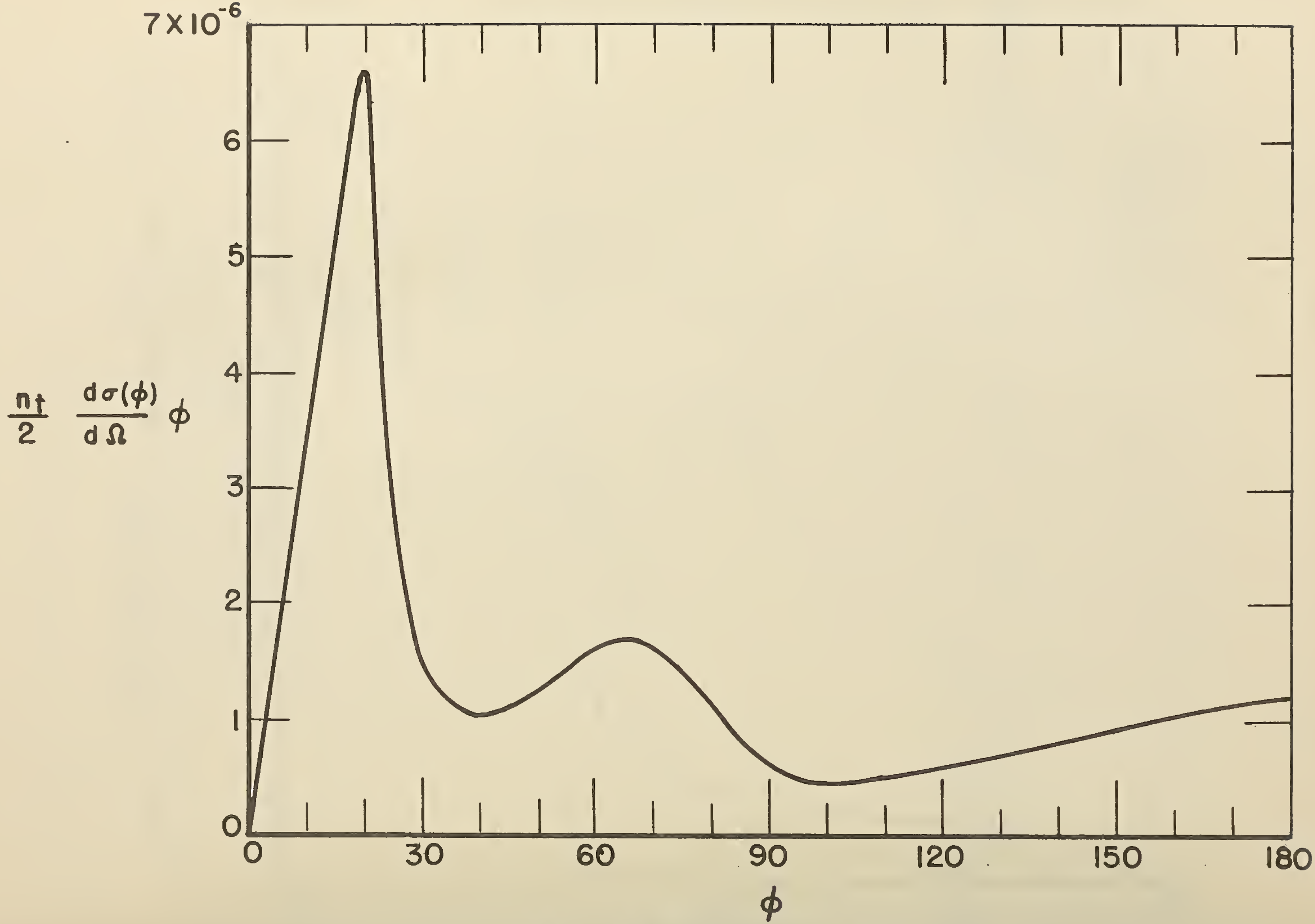

Figure 3. Number of single scattered neutrons as a function of scattering angle.

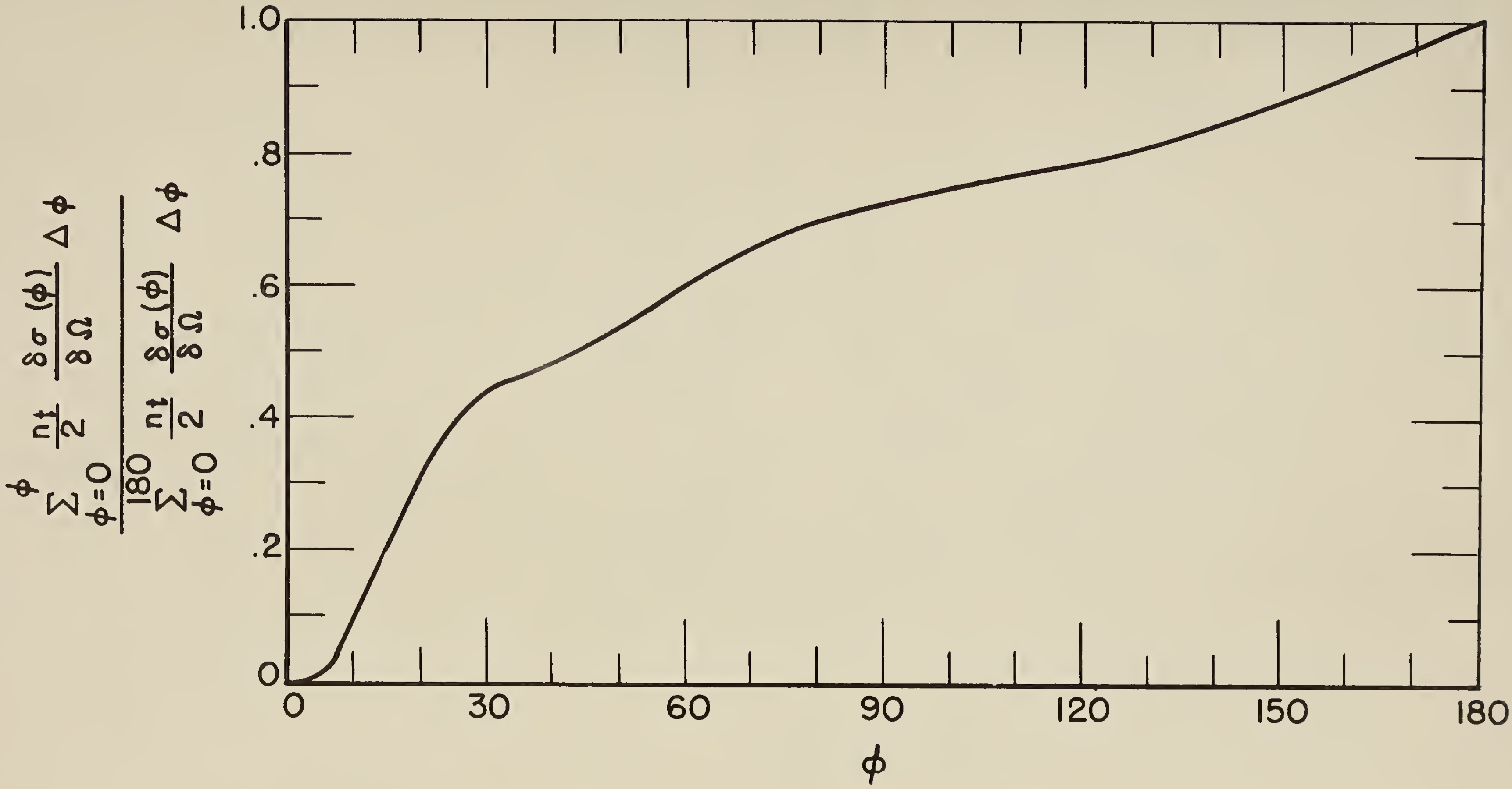

Figure 4. Relative number of neutrons reaching the detector with scattering angles less than a given angle.

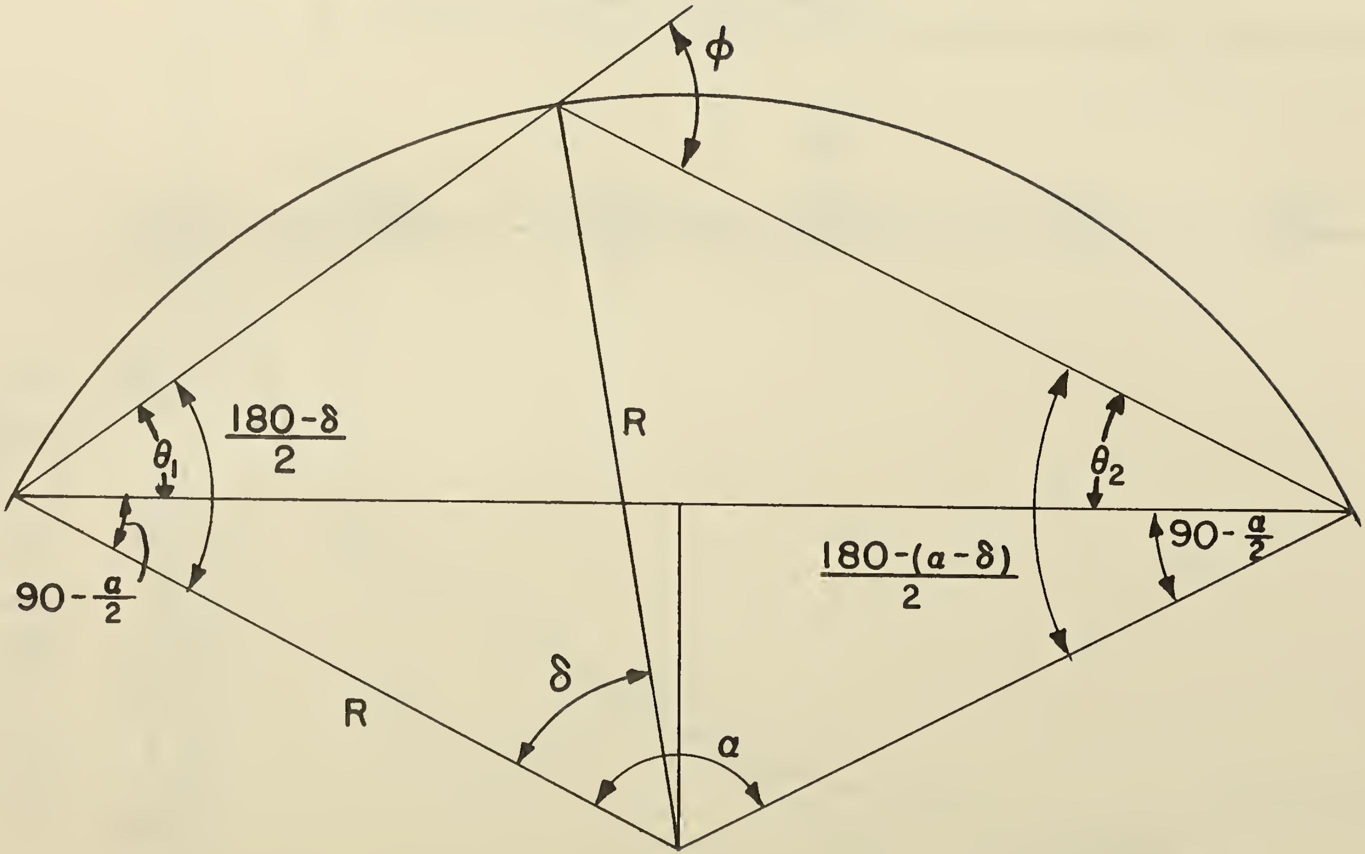

Figure 5. Geometry for note.

U.S. DEPARTMENT OF COMMERCE
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